

Quick Announcements

- Gradescope is now live for HW1. Find the link for it from Canvas.
- Upload a requirements.txt file to Gradescope if your code needs non-standard packages.
- Office hours on the course website have been updated

Architecture Advancement on Transformers

Large Language Models: Methods and Applications

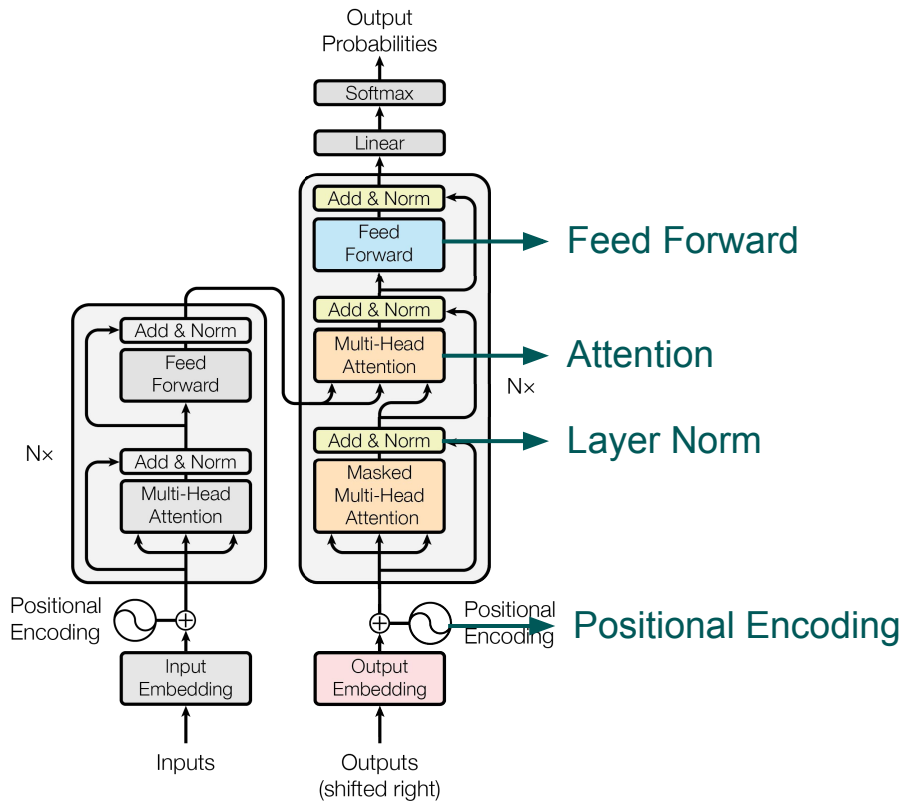
Daphne Ippolito and Chenyan Xiong

Learning Objectives

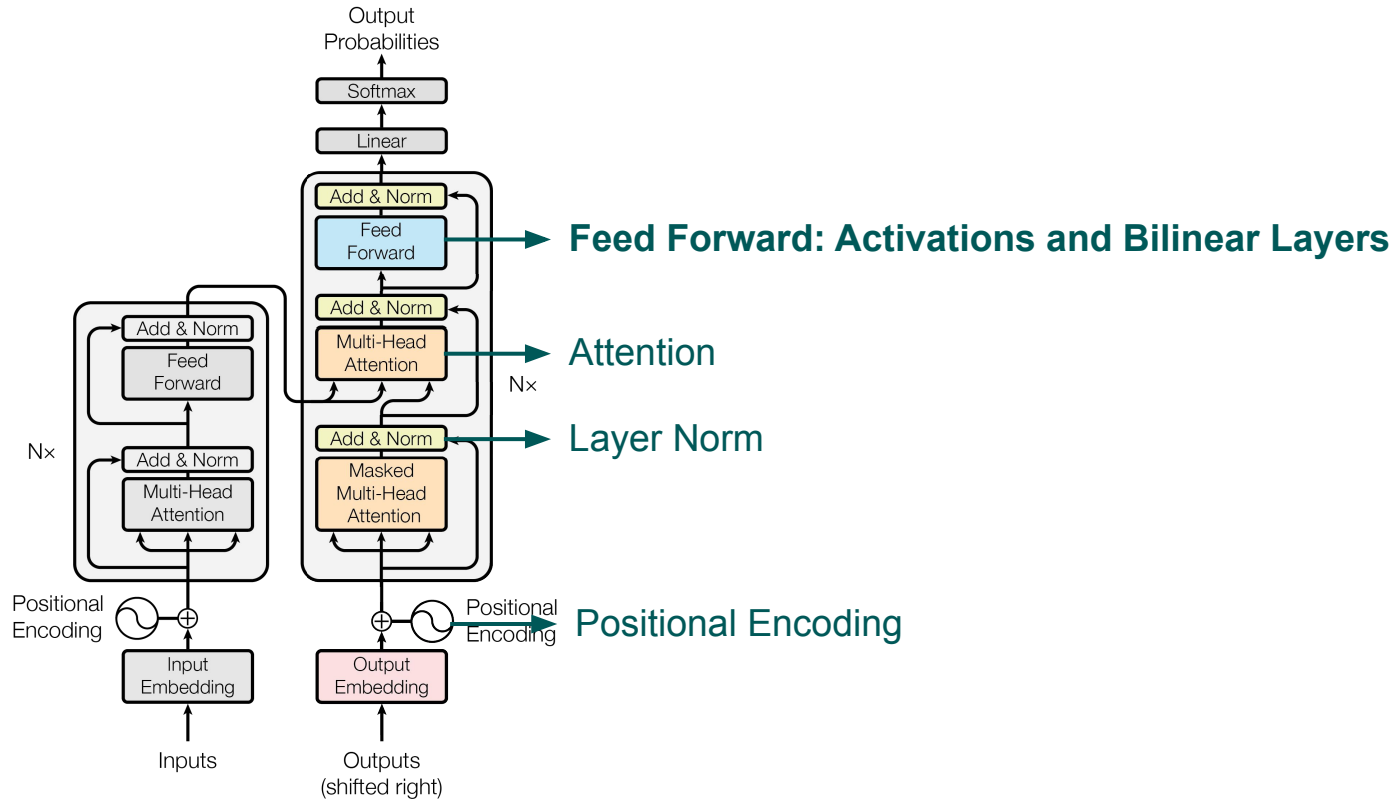
An overview of recent architecture advancements on top of Transformers

- A clear grasp on the details of new architectures
- Understand the motivation and benefits of each architecture upgrades
- Apply the right architecture specifications for target scenarios
- [Optional] Explore new architecture designs in your research

Places for Improvements



Places for Improvements



Feed Forward: Activations and Bilinear Layers

Variants of linear FFN Layers (omitting bias):

$$\text{FFN}_{\text{RELU}}(x) = \text{RELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{RELU}(x\mathbf{W}_1) = \max(0, x\mathbf{W}_1)\mathbf{W}_2$$

Feed Forward: Activations and Bilinear Layers

Variants of linear FFN Layers (omitting bias):

$$\text{FFN}_{\text{RELU}}(x) = \text{RELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{RELU}(x\mathbf{W}_1) = \max(0, x\mathbf{W}_1)\mathbf{W}_2$$

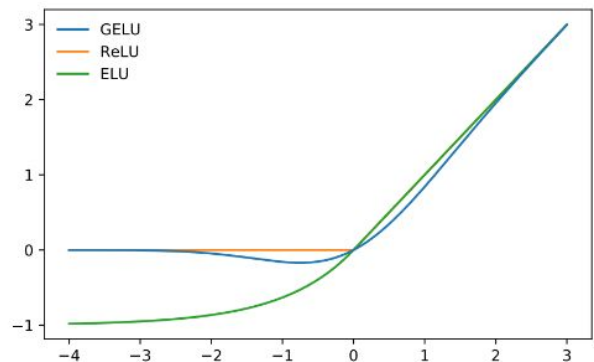
$$\text{FFN}_{\text{GELU}}(x) = \text{GELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{GELU}(x\mathbf{W}_1) = xP(X < x) = x\Phi(x)$$

Feed Forward: Activations and Bilinear Layers

Variants of linear FFN Layers (omitting bias):

$$\text{FFN}_{\text{RELU}}(x) = \text{RELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{RELU}(x\mathbf{W}_1) = \max(0, x\mathbf{W}_1)\mathbf{W}_2$$

$$\text{FFN}_{\text{GELU}}(x) = \text{GELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{GELU}(x\mathbf{W}_1) = xP(X < x) = x\Phi(x)$$



GELU Activation [1]

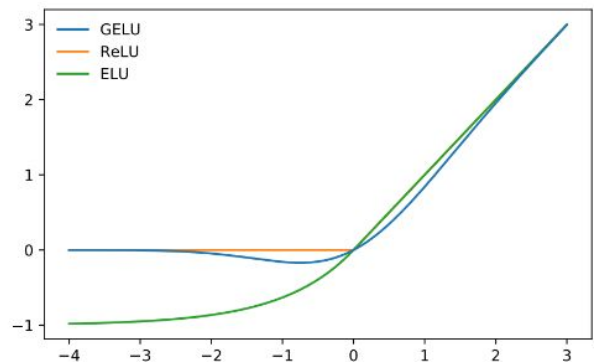
Feed Forward: Activations and Bilinear Layers

Variants of linear FFN Layers (omitting bias):

$$\text{FFN}_{\text{RELU}}(x) = \text{RELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{RELU}(x\mathbf{W}_1) = \max(0, x\mathbf{W}_1)\mathbf{W}_2$$

$$\text{FFN}_{\text{GELU}}(x) = \text{GELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{GELU}(x\mathbf{W}_1) = xP(X < x) = x\Phi(x)$$

$$\text{FFN}_{\text{Switch}}(x) = \text{Swish}_1(x\mathbf{W}_1)\mathbf{W}_2; \text{Swish}_\beta(x\mathbf{W}_1) = x\text{Sigmod}(\beta x)$$



GELU Activation [1]

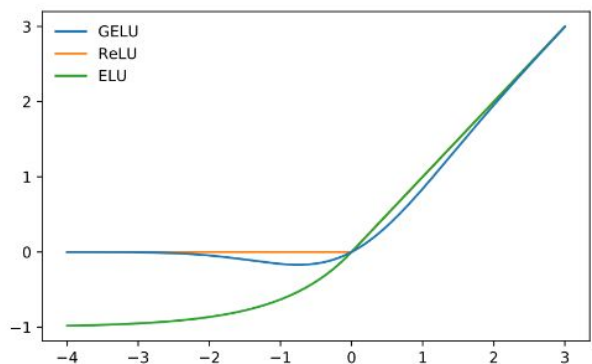
Feed Forward: Activations and Bilinear Layers

Variants of linear FFN Layers (omitting bias):

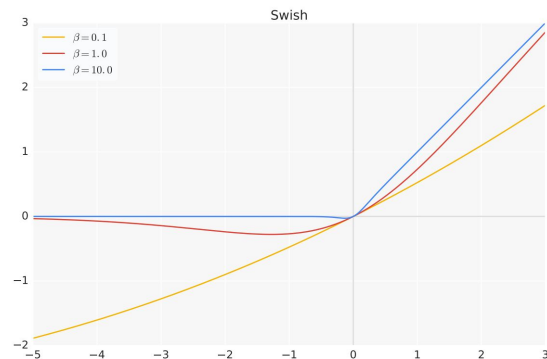
$$\text{FFN}_{\text{RELU}}(x) = \text{RELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{RELU}(x\mathbf{W}_1) = \max(0, x\mathbf{W}_1)\mathbf{W}_2$$

$$\text{FFN}_{\text{GELU}}(x) = \text{GELU}(x\mathbf{W}_1)\mathbf{W}_2; \text{GELU}(x\mathbf{W}_1) = xP(X < x) = x\Phi(x)$$

$$\text{FFN}_{\text{Switch}}(x) = \text{Swish}_1(x\mathbf{W}_1)\mathbf{W}_2; \text{Swish}_\beta(x\mathbf{W}_1) = x\text{Sigmod}(\beta x)$$



GELU Activation [1]



Switch Activation [2]

Feed Forward: Activations and Bilinear Layers

Bilinear FFNs (Omitting Bias):

$\text{FFN}_{\text{Bilinear}}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$. Two FFN with componentwise product

Feed Forward: Activations and Bilinear Layers

Bilinear FFNs (Omitting Bias):

$\text{FFN}_{\text{Bilinear}}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$. Two FFN with componentwise product

$\text{FFN}_{\text{ReLU}}(x) = (\text{ReLU}(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding RELU activation on one FFN

$\text{FFN}_{\text{GELU}}(x) = (\text{GELU}(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding GELU activation on one FFN

$\text{FFN}_{\text{SwiGLU}}(x) = (\text{Swish}_1(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding Swish activation on one FFN

Feed Forward: Activations and Bilinear Layers

Bilinear FFNs (Omitting Bias):

$\text{FFN}_{\text{Bilinear}}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$. Two FFN with componentwise product

$\text{FFN}_{\text{ReLU}}(x) = (\text{RELU}(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding RELU activation on one FFN

$\text{FFN}_{\text{GELU}}(x) = (\text{GELU}(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding GELU activation on one FFN

$\text{FFN}_{\text{SwiGLU}}(x) = (\text{Swish}_1(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding Swish activation on one FFN

With reduced hidden dimension of the projections to keep parameter count the same

Training Steps	65,536	524,288	
$\text{FFN}_{\text{ReLU}}(\textit{baseline})$	1.997 (0.005)	1.677	
FFN_{GELU}	1.983 (0.005)	1.679	
$\text{FFN}_{\text{Swish}}$	1.994 (0.003)	1.683	
FFN_{GLU}	1.982 (0.006)	1.663	
$\text{FFN}_{\text{Bilinear}}$	1.960 (0.005)	1.648	
$\text{FFN}_{\text{GEGLU}}$	1.942 (0.004)	1.633	Improved speed-quality
$\text{FFN}_{\text{SwiGLU}}$	1.944 (0.010)	1.636	
FFN_{ReLU}	1.953 (0.003)	1.645	

T5 base Perplexity at Pretraining Steps [3]

Feed Forward: Activations and Bilinear Layers

Bilinear FFNs (Omitting Bias):

$\text{FFN}_{\text{Bilinear}}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$. Two FFN with componentwise product

$\text{FFN}_{\text{ReLU}}(x) = (\text{ReLU}(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding RELU activation on one FFN

$\text{FFN}_{\text{GELU}}(x) = (\text{GELU}(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding GELU activation on one FFN

$\text{FFN}_{\text{SwiGLU}}(x) = (\text{Swish}_1(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding Swish activation on one FFN

With reduced hidden dimension of the projections to keep parameter count the same

Training Steps	65,536	524,288
$\text{FFN}_{\text{ReLU}}(\text{baseline})$	1.997 (0.005)	1.677
FFN_{GELU}	1.983 (0.005)	1.679
$\text{FFN}_{\text{Swish}}$	1.994 (0.003)	1.683
FFN_{GLU}	1.982 (0.006)	1.663
$\text{FFN}_{\text{Bilinear}}$	1.960 (0.005)	1.648
$\text{FFN}_{\text{GEGLU}}$	1.942 (0.004)	1.633
$\text{FFN}_{\text{SwiGLU}}$	1.944 (0.010)	1.636
FFN_{ReLU}	1.953 (0.003)	1.645

**Improved
speed-quality**

“We offer no explanation as to why these architectures seem to work; we attribute their success, as all else, to divine benevolence”---Noam Shazeer. 2017

T5 base Perplexity at Pretraining Steps [3]

Feed Forward: Activations and Bilinear Layers

Bilinear FFNs (Omitting Bias):

$\text{FFN}_{\text{Bilinear}}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$. Two FFN with componentwise product

$\text{FFN}_{\text{ReLU}}(x) = (\text{RELU}(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding RELU activation on one FFN

$\text{FFN}_{\text{GEGLU}}(x) = (\text{GELU}(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding GELU activation on one FFN

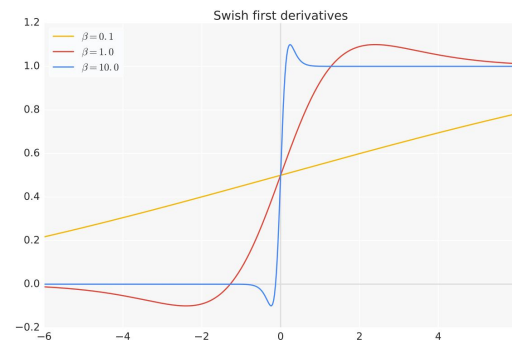
$\text{FFN}_{\text{SwiGLU}}(x) = (\text{Swish}_1(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$. Adding Swish activation on one FFN

With reduced hidden dimension of the projections to keep parameter count the same

Training Steps	65,536	524,288
$\text{FFN}_{\text{ReLU}}(\text{baseline})$	1.997 (0.005)	1.677
FFN_{GELU}	1.983 (0.005)	1.679
$\text{FFN}_{\text{Swish}}$	1.994 (0.003)	1.683
FFN_{GLU}	1.982 (0.006)	1.663
$\text{FFN}_{\text{Bilinear}}$	1.960 (0.005)	1.648
$\text{FFN}_{\text{GEGLU}}$	1.942 (0.004)	1.633
$\text{FFN}_{\text{SwiGLU}}$	1.944 (0.010)	1.636
FFN_{ReLU}	1.953 (0.003)	1.645

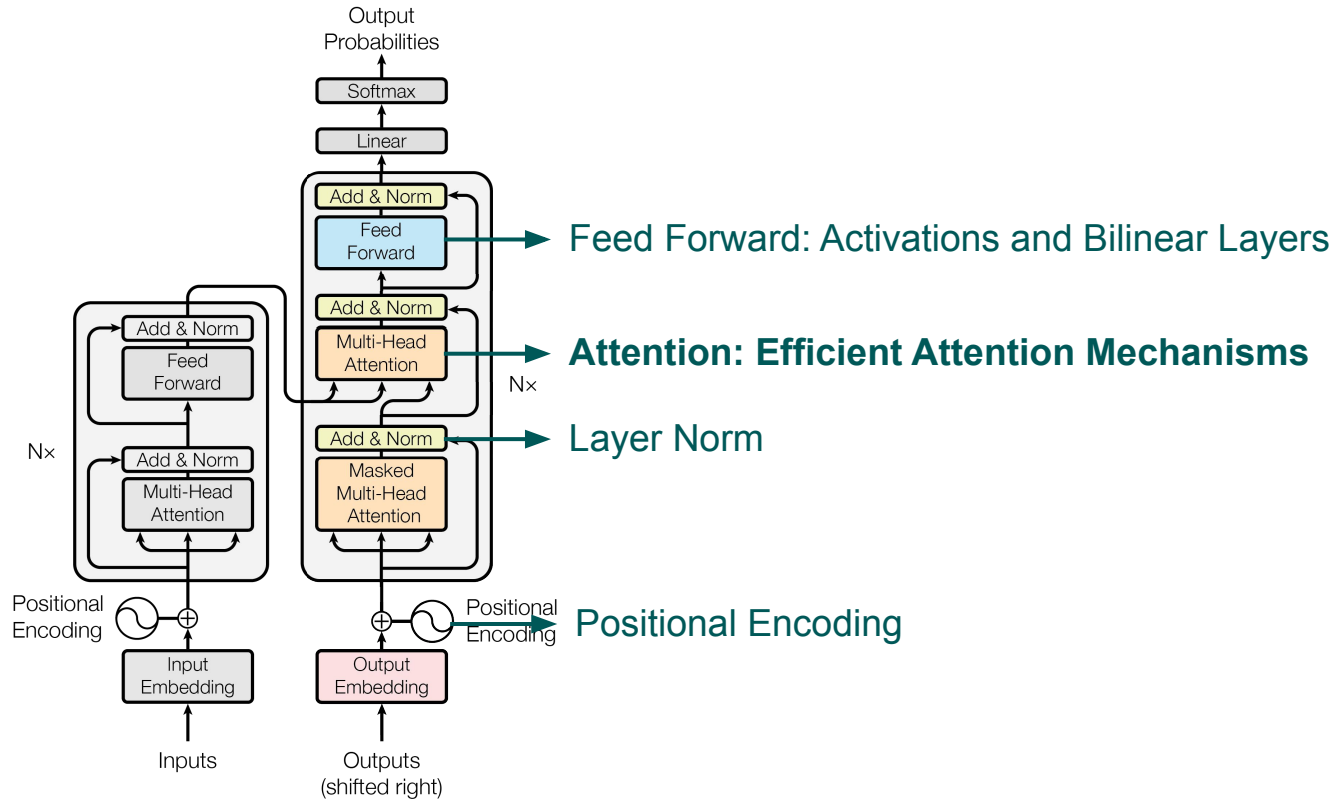
Improved
speed-quality

T5 base Perplexity at Pretraining Steps [3]



Swish Gradients [2]

Places for Improvements



Attention: Efficient Attention Mechanisms

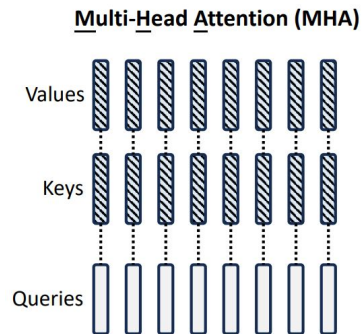
Standard Multi-Head Attention

$$\text{head}_1 = \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V)$$

$\vdots$

$$\text{head}_H = \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_H^K, \mathbf{VW}_H^V)$$

$$\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \\ \text{Concat}(\text{head}_1, \dots, \text{head}_H)$$



Attention: Efficient Attention Mechanisms

Standard Multi-Head Attention

$$\begin{aligned}\text{head}_1 &= \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V) \\ &\vdots \\ \text{head}_H &= \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_H^K, \mathbf{VW}_H^V)\end{aligned}$$

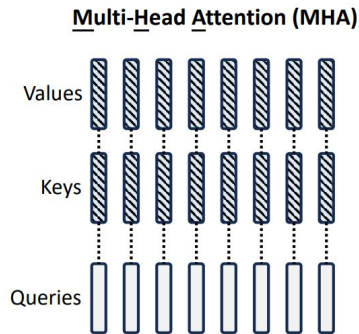
$$\begin{aligned}\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) &= \\ &\text{Concat}(\text{head}_1, \dots, \text{head}_H)\end{aligned}$$

**Inputs and outputs of each layer
are the same dimensions:**

$$\mathbf{Q} \in \mathbb{R}^{T \times d_{\text{model}}}$$

$$\mathbf{K} \in \mathbb{R}^{T \times d_{\text{model}}}$$

$$\mathbf{V} \in \mathbb{R}^{T \times d_{\text{model}}}$$



Attention: Efficient Attention Mechanisms

Standard Multi-Head Attention

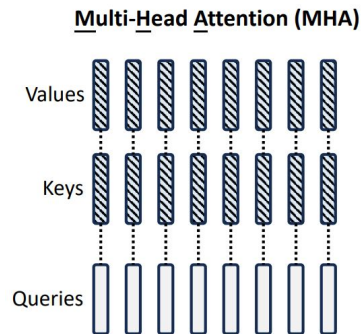
$$\begin{aligned}\text{head}_1 &= \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V) \\ &\vdots \\ \text{head}_H &= \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_H^K, \mathbf{VW}_H^V)\end{aligned}$$

$$\begin{aligned}\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) &= \\ &\text{Concat}(\text{head}_1, \dots, \text{head}_H)\end{aligned}$$

**Inputs and outputs of each layer
are the same dimensions:**

$$\begin{aligned}\mathbf{Q} &\in \mathbb{R}^{T \times d_{\text{model}}} \\ \mathbf{K} &\in \mathbb{R}^{T \times d_{\text{model}}} \\ \mathbf{V} &\in \mathbb{R}^{T \times d_{\text{model}}}\end{aligned}$$

Huge memory consumption during inference. Needs to keep one K, V for each layer and each position



Attention: Efficient Attention Mechanisms

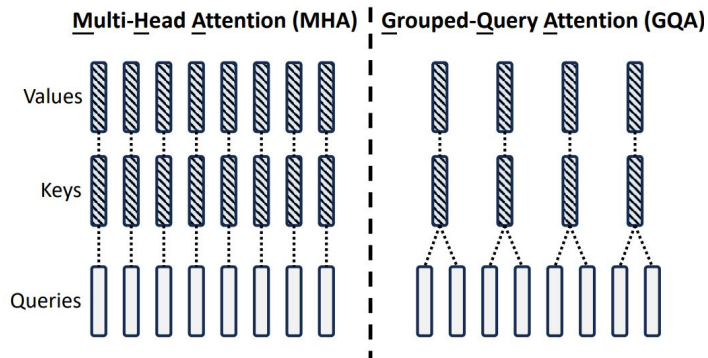
Grouped-Query Attention: Divide Q in G groups, and share K, V in the same group [4]

$$\text{head}_1 = \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V)$$

$\vdots$

$$\text{head}_H = \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_G^K, \mathbf{VW}_G^V)$$

$$\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \\ \text{Concat}(\text{head}_1, \dots, \text{head}_H)$$



Attention: Efficient Attention Mechanisms

Grouped-Query Attention: Divide Q in G groups, and share K, V in the same group [4]

$$\text{head}_1 = \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V)$$

$\vdots$

$$\text{head}_H = \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_G^K, \mathbf{VW}_G^V)$$

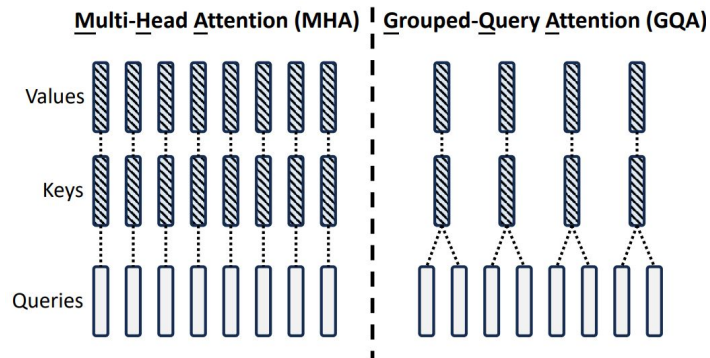
$$\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \text{Concat}(\text{head}_1, \dots, \text{head}_H)$$

**Inputs and outputs of each layer
are the same dimensions:**

$$\mathbf{Q} \in \mathbb{R}^{T \times d_{\text{model}}}$$

$$\mathbf{K} \in \mathbb{R}^{T/(\frac{H}{G}) \times d_{\text{model}}}$$

$$\mathbf{V} \in \mathbb{R}^{T/(\frac{H}{G}) \times d_{\text{model}}}$$



Attention: Efficient Attention Mechanisms

Grouped-Query Attention: Divide Q in G groups, and share K, V in the same group [4]

$$\text{head}_1 = \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V)$$

$\vdots$

$$\text{head}_H = \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_G^K, \mathbf{VW}_G^V)$$

$$\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \text{Concat}(\text{head}_1, \dots, \text{head}_H)$$

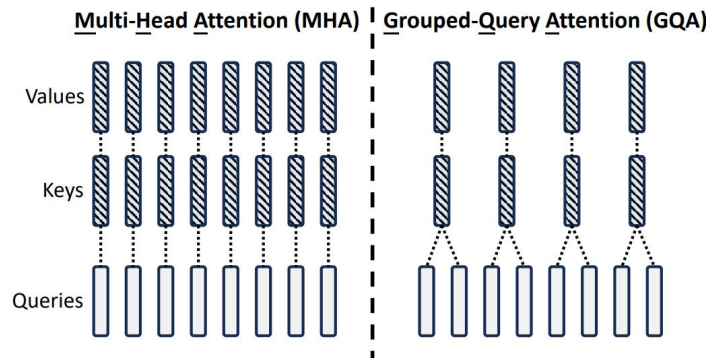
**Inputs and outputs of each layer
are the same dimensions:**

$$\mathbf{Q} \in \mathbb{R}^{T \times d_{\text{model}}}$$

$$\mathbf{K} \in \mathbb{R}^{T/(\frac{H}{G}) \times d_{\text{model}}}$$

$$\mathbf{V} \in \mathbb{R}^{T/(\frac{H}{G}) \times d_{\text{model}}}$$

Reduce K, V cache storage
to group sizes



Attention: Efficient Attention Mechanisms

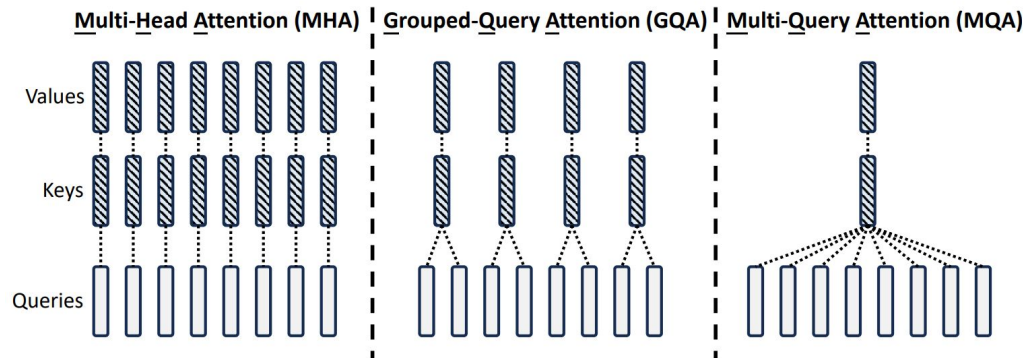
Multi-Query Attention: Single K, V for all Q heads [5]

$$\begin{aligned}\text{head}_1 &= \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V) \\ &\vdots \\ \text{head}_H &= \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V)\end{aligned}$$

$$\begin{aligned}\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) &= \\ &\text{Concat}(\text{head}_1, \dots, \text{head}_H)\end{aligned}$$

**Inputs and outputs of each layer
are the same dimensions:**

$$\begin{aligned}\mathbf{Q} &\in \mathbb{R}^{T \times d_{\text{model}}} \\ \mathbf{K} &\in \mathbb{R}^{T/H \times d_{\text{model}}} \\ \mathbf{V} &\in \mathbb{R}^{T/H \times d_{\text{model}}}\end{aligned}$$



Attention: Efficient Attention Mechanisms

Multi-Query Attention: Single K, V for all Q heads [5]

$$\begin{aligned}\text{head}_1 &= \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V) \\ &\vdots \\ \text{head}_H &= \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V)\end{aligned}$$

$$\begin{aligned}\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) &= \\ &\text{Concat}(\text{head}_1, \dots, \text{head}_H)\end{aligned}$$

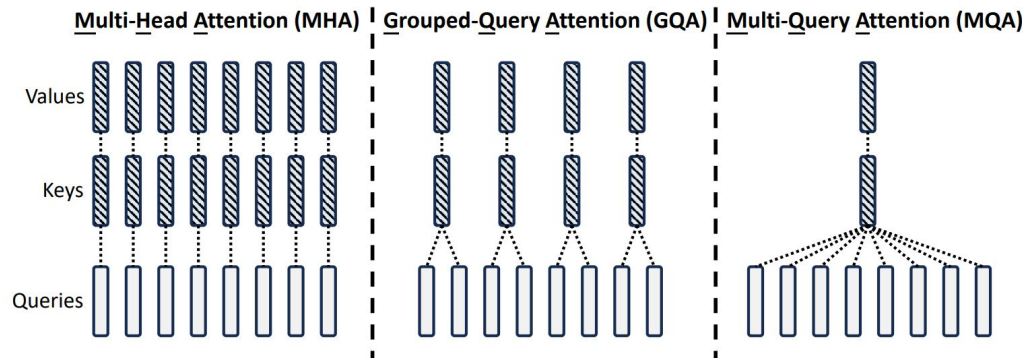
**Inputs and outputs of each layer
are the same dimensions:**

$$\mathbf{Q} \in \mathbb{R}^{T \times d_{\text{model}}}$$

$$\mathbf{K} \in \mathbb{R}^{T/H \times d_{\text{model}}}$$

$$\mathbf{V} \in \mathbb{R}^{T/H \times d_{\text{model}}}$$

→ Further Reduce K, V Cache Size



Attention: Efficient Attention Mechanisms

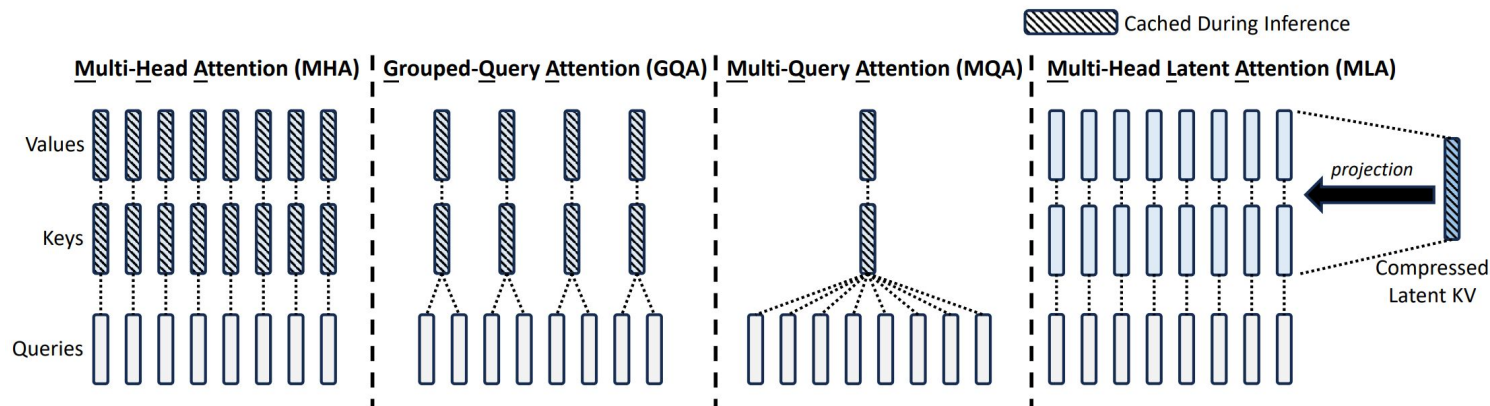
Multi-Head Latent Attention: Project K, V into a lower dimension latent vector [6]

$$\mathbf{k}_t = \mathbf{W}^{UK} \mathbf{c}_t^{KV}$$

$$\mathbf{v}_t = \mathbf{W}^{UV} \mathbf{c}_t^{KV}$$

$$\mathbf{c}_t^{KV} = \mathbf{W}^{DKV} \mathbf{h}_t \quad \text{Only latent vector to store}$$

Recovery of k, v can be merged
with q in attention layer operations



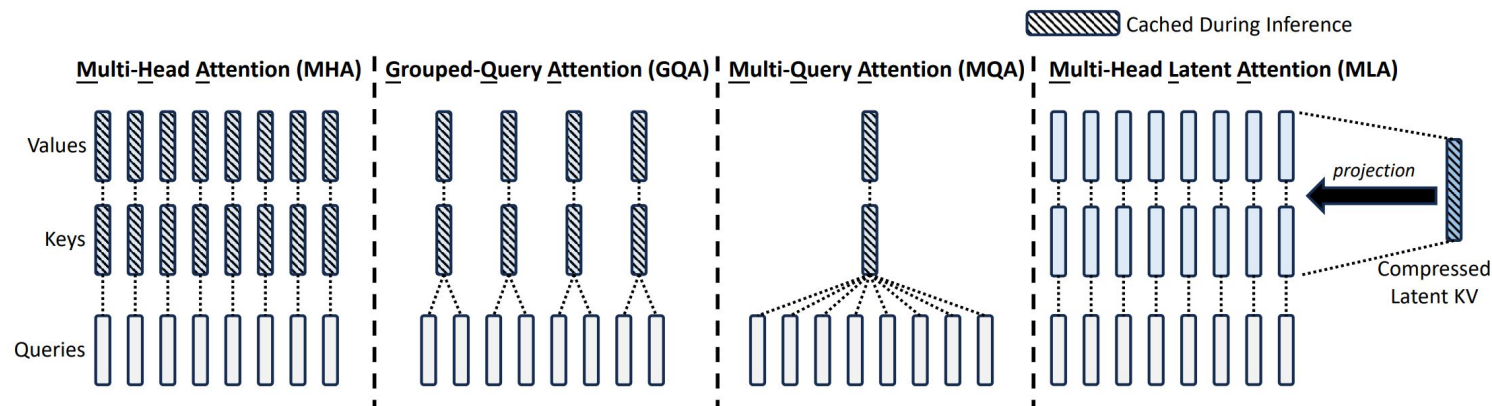
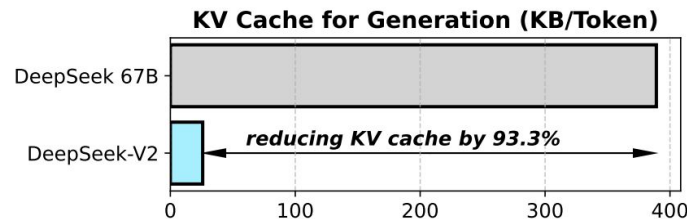
Attention: Efficient Attention Mechanisms

Multi-Head Latent Attention: Project K, V into a lower dimension latent vector [6]

$$k_t = W^{UK} c_t^{KV}$$

$$v_t = W^{UV} c_t^{KV}$$

$$c_t^{KV} = W^{DKV} h_t \quad \text{Only latent vector to store}$$



Attention: Efficient Attention Mechanisms

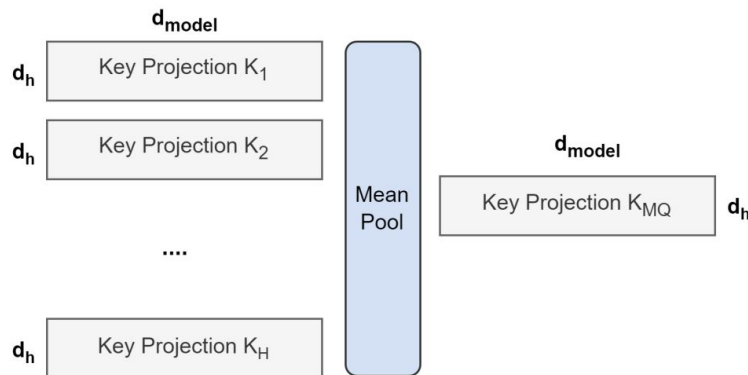
How to use efficient attention mechanisms:

- Pretraining directly with updated architecture
- Or use it as a compression method to speed up a rich multi-head attention model

Attention: Efficient Attention Mechanisms

How to use efficient attention mechanisms:

- Pretraining directly with updated architecture
- Or use it as a compression method to speed up a rich multi-head attention model

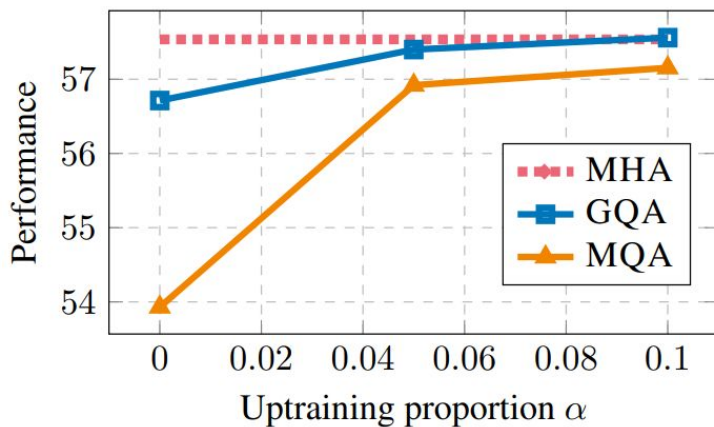


**Mean Pooling Multi-Head Attention to
Grouped-Query Attention [4]**

Attention: Efficient Attention Mechanisms

Performance:

- Recovering similar effectiveness as multi-head attention
- Significantly improve generation speed

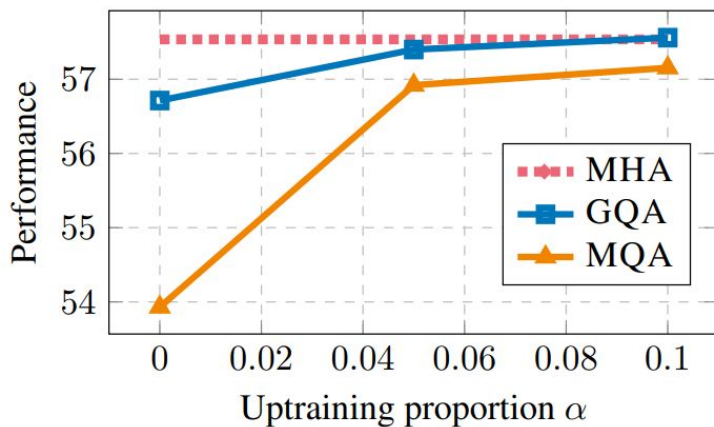


**Performance of Grouped-Query Attention
Adapted from Multi-Head Attention [4]**

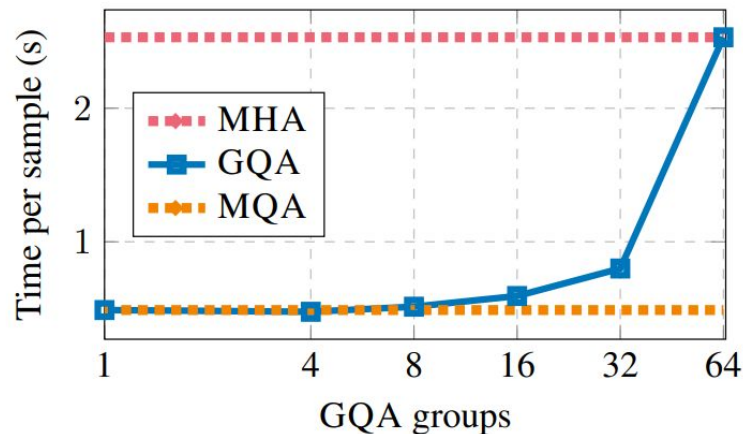
Attention: Efficient Attention Mechanisms

Performance:

- Recovering similar effectiveness as multi-head attention
- Significantly improve generation speed

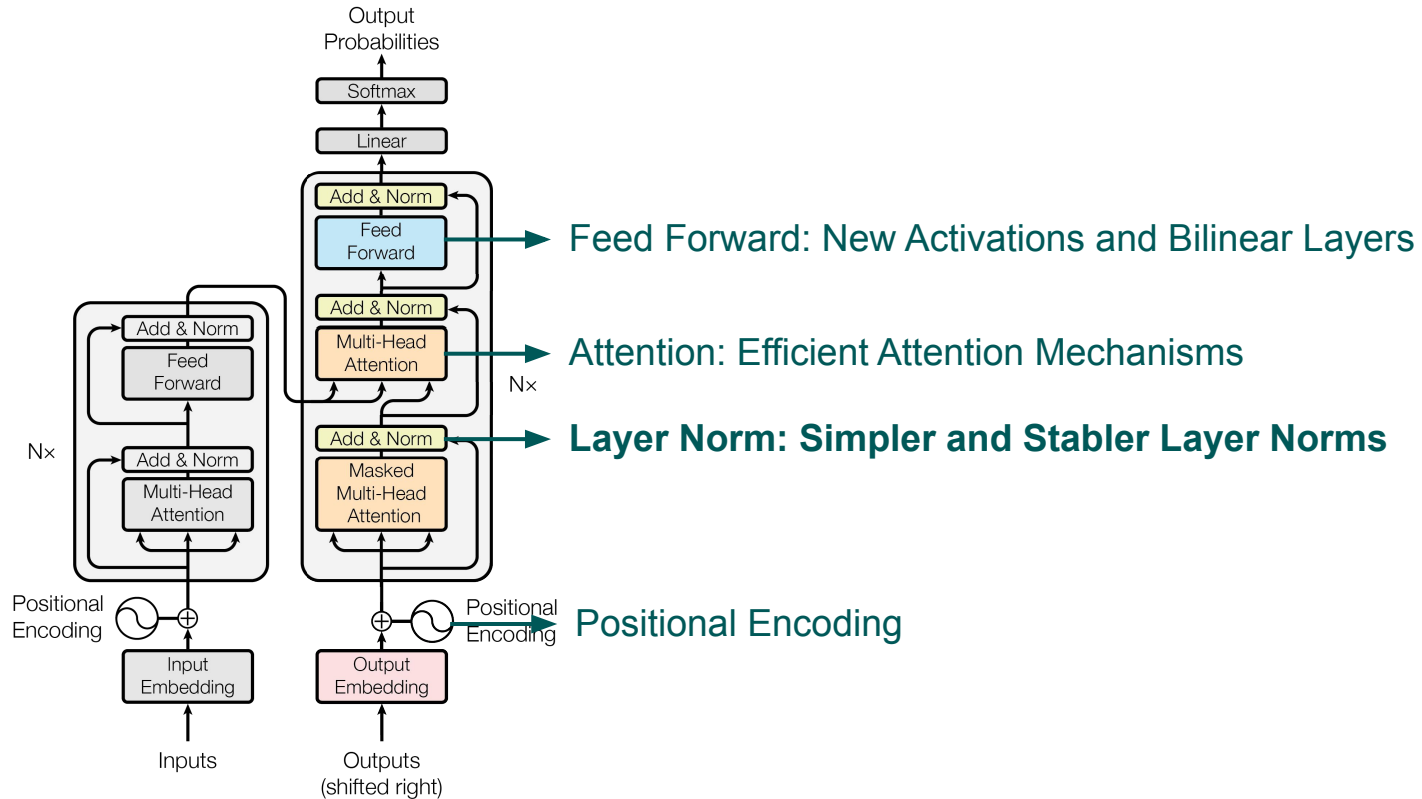


**Performance of Grouped-Query Attention
Adapted from Multi-Head Attention [4]**



**Generation Efficiency with Grouped-Query
Attention [4]**

Places for Improvements



Layernorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

$$\text{LN}(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Learnable parameter
started from 1

Align the outputs to standard distributions to
improve training convergence and stability

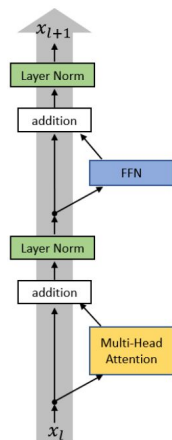
Layernorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

$$\text{LN}(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Learnable parameter
started from 1

Align the outputs to standard distributions to
improve training convergence and stability



Post-LN: After
Residual Sum

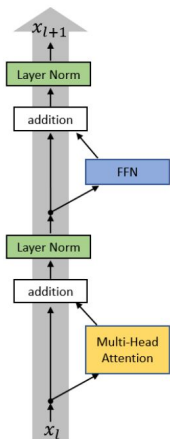
Layernorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

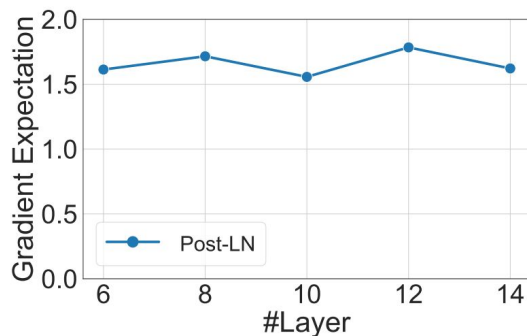
$$\text{LN}(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Learnable parameter
started from 1

Align the outputs to standard distributions to
improve training convergence and stability



Post-LN: After
Residual Sum



Large gradient L2 norm, leading to
unstable training at large scale [6]

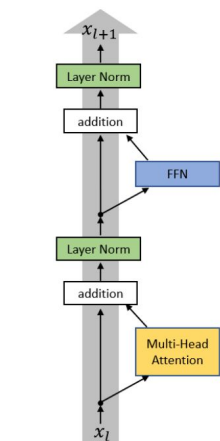
LayerNorm: Simpler and Stabler LayerNorms

The Layer Normalization Layer [5]:

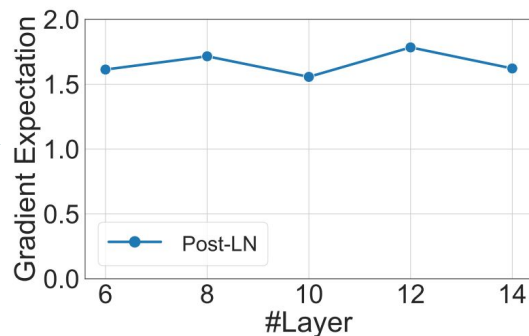
$$\text{LN}(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Learnable parameter
started from 1

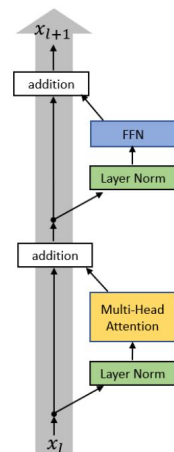
Align the outputs to standard distributions to
improve training convergence and stability



Post-LN: After
Residual Sum



Large gradient L2 norm, leading to
unstable training at large scale [6]



Pre-LN: Before
Residual Sum

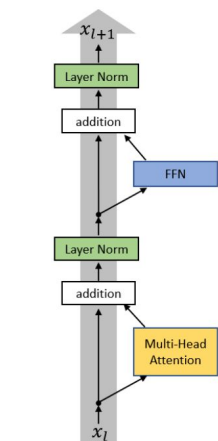
Layernorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

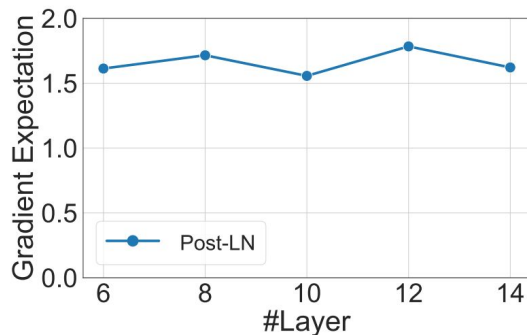
$$\text{LN}(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Learnable parameter
started from 1

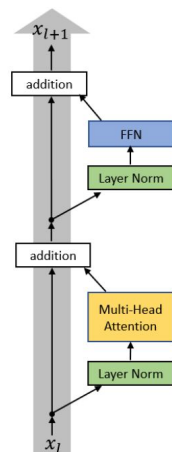
Align the outputs to standard distributions to
improve training convergence and stability



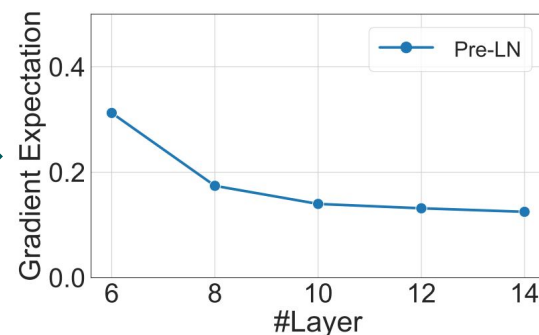
Post-LN: After
Residual Sum



Large gradient L2 norm, leading to
unstable training at large scale [6]



Pre-LN: Before
Residual Sum



Lower gradient L2 norm, improved
training stability [6]

LayerNorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

$$\text{LN}(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Layernorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

$$\text{LN}(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

RMSNorm: Only rescaling, no recentering [7]

$$\text{LN}(x_i) = \frac{x_i}{\text{RMS}(\mathbf{x})} g_i; \quad \text{RMS}(\mathbf{x}) = \sqrt{\frac{1}{d} \sum_i (x_i)^2}$$

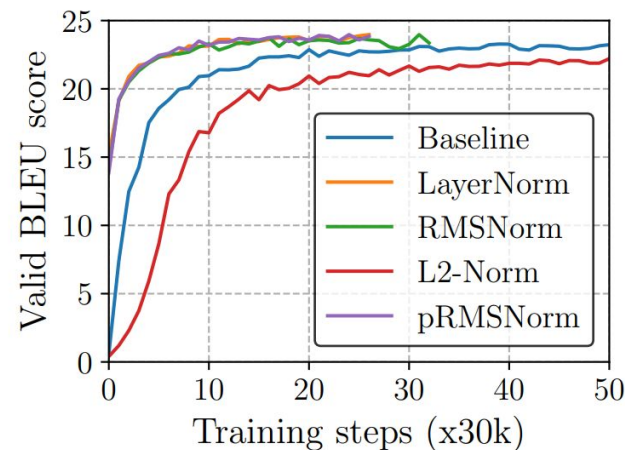
Layernorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

$$\text{LN}(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

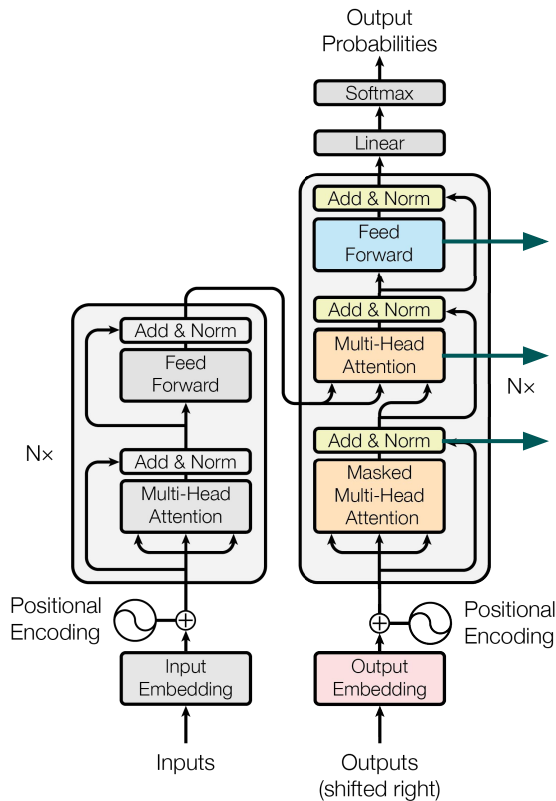
RMSNorm: Only rescaling, no recentering [7]

$$\text{LN}(x_i) = \frac{x_i}{\text{RMS}(\mathbf{x})} g_i; \quad \text{RMS}(\mathbf{x}) = \sqrt{\frac{1}{d} \sum_i (x_i)^2}$$



Better Convergence Rate on Machine Translation and Many NLP Tasks [7]

Places for Improvements: Recap



These architecture upgrades are adapted in most recent public production LLMs



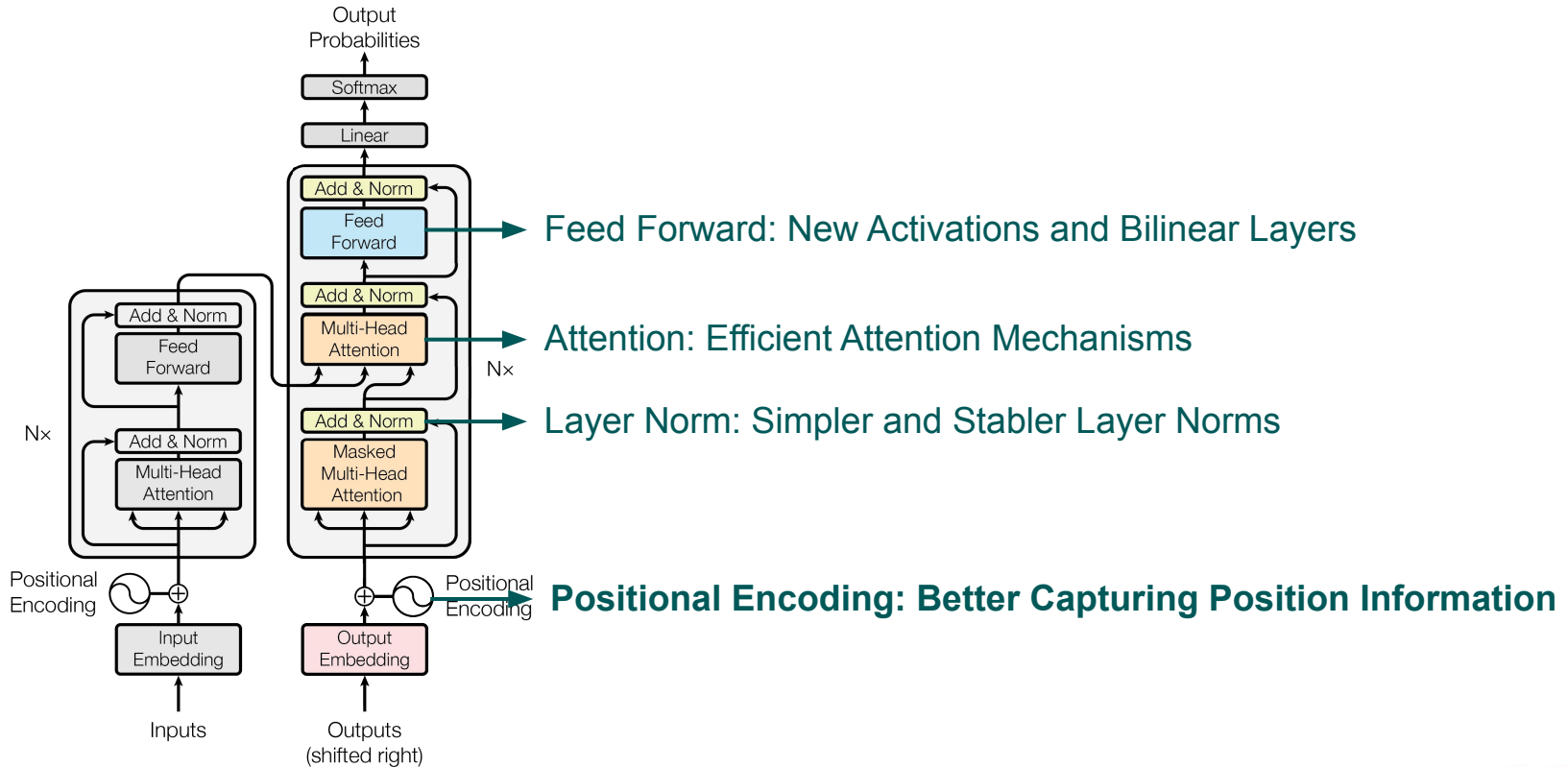
Most Transformer architecture upgrades are for efficiency and large-scale learning stability. Simplicity is often the winner.



Most Transformer architecture upgrades are for efficiency and large-scale learning stability. Simplicity is often the winner.

Why?

Places for Improvements

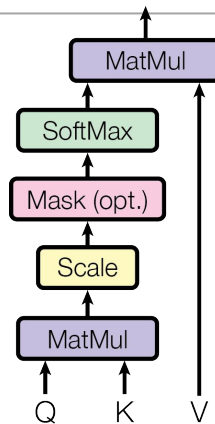


Position Encoding: Capturing Position Information

Transformer itself is position agnostic:

$$\text{attention output at position } j = \sum_{i=1}^T \text{score}(\mathbf{q}_j, \mathbf{k}_i) \cdot \mathbf{v}_i$$

$$\text{score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$



Position Encoding: Capturing Position Information

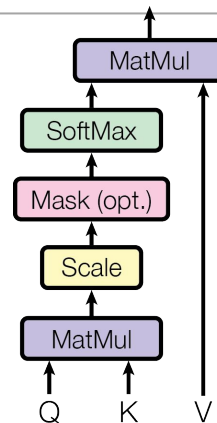
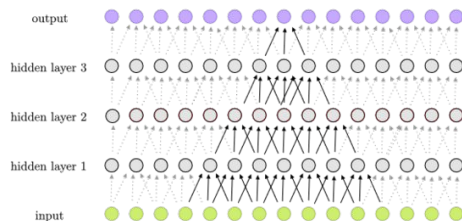
Transformer itself is position agnostic:

$$\text{attention output at position } j = \sum_{i=1}^T \text{score}(\mathbf{q}_j, \mathbf{k}_i) \cdot \mathbf{v}_i$$

$$\text{score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$

In comparison

CNN has locality prior:



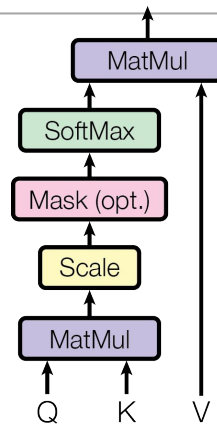
Position Encoding: Capturing Position Information

Transformer itself is position agnostic:

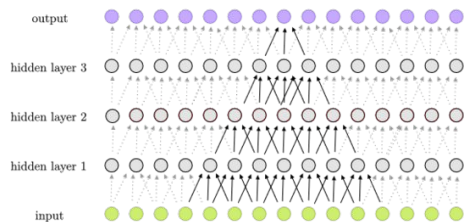
$$\text{attention output at position } j = \sum_{i=1}^T \text{score}(\mathbf{q}_j, \mathbf{k}_i) \cdot \mathbf{v}_i$$

$$\text{score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$

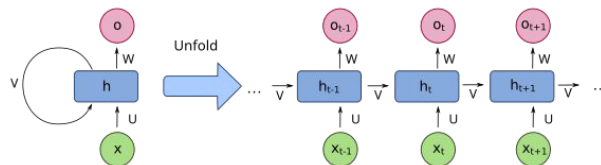
In comparison



CNN has locality prior:



RNN has sequential prior:



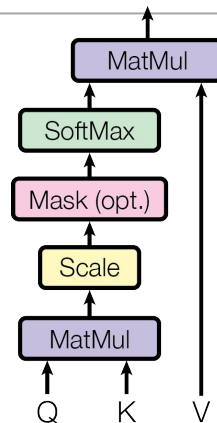
Position Encoding: Capturing Position Information

Transformer itself is position agnostic:

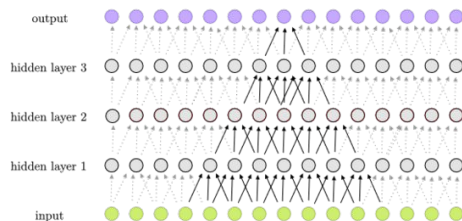
$$\text{attention output at position } j = \sum_{i=1}^T \text{score}(\mathbf{q}_j, \mathbf{k}_i) \cdot \mathbf{v}_i$$

$$\text{score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$

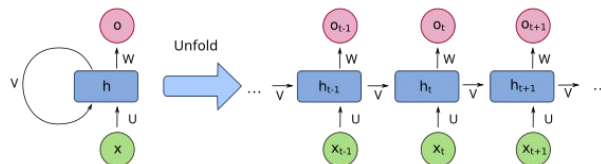
In comparison



CNN has locality prior:



RNN has sequential prior:



Position encoding adds positional information which is useful for language

Position Encoding: Absolute Position Information

Additive Position Encoding: add the positional information in the token embedding layer

$$\mathbf{x}' = \mathbf{x} + \mathbf{p}_{pos}$$

Position Embedding at position pos

Position Encoding: Absolute Position Information

Additive Position Encoding: add the positional information in the token embedding layer

$$\mathbf{x}' = \mathbf{x} + \mathbf{p}_{pos}$$

Position Embedding at position pos

Sinusoid position embedding

$$p_{pos,2i} = \sin(pos/10000^{2i/d})$$

$$p_{pos,2i+1} = \cos(pos/10000^{2i/d})$$

Position Encoding: Absolute Position Information

Additive Position Encoding: add the positional information in the token embedding layer

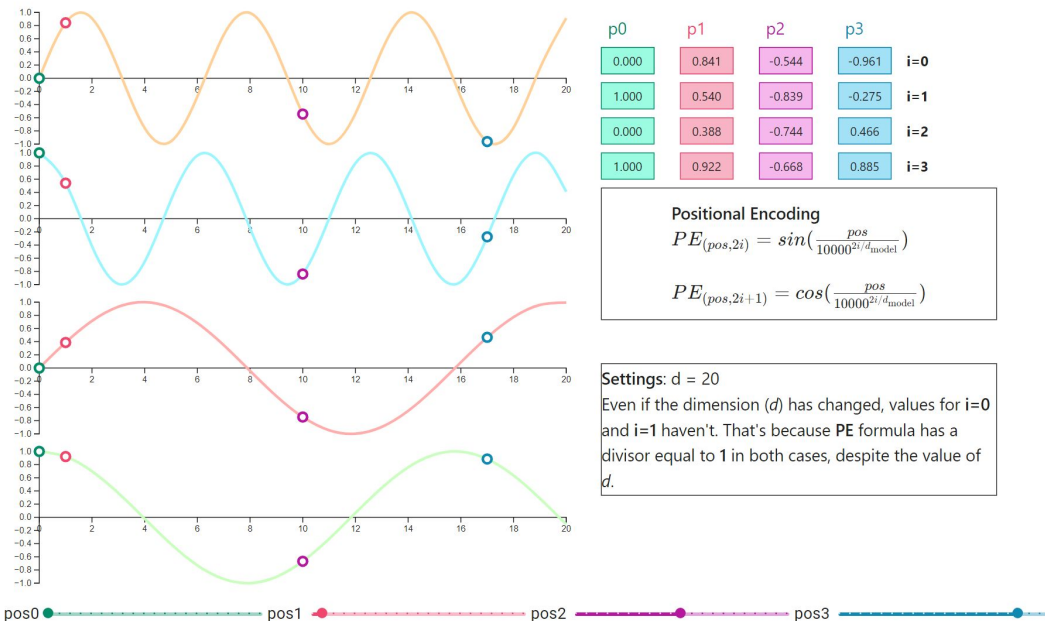
$$\mathbf{x}' = \mathbf{x} + \mathbf{p}_{pos}$$

Position Embedding at position pos

Sinusoid position embedding

$$p_{pos,2i} = \sin(pos/10000^{2i/d})$$

$$p_{pos,2i+1} = \cos(pos/10000^{2i/d})$$



<https://erdem.pl/2021/05/understanding-positional-encoding-in-transformers>

Position Encoding: Absolute Position Information

Additive Position Encoding: add the positional information in the token embedding layer

$$\mathbf{x}' = \mathbf{x} + \mathbf{p}_{pos}$$

Position Embedding at position pos

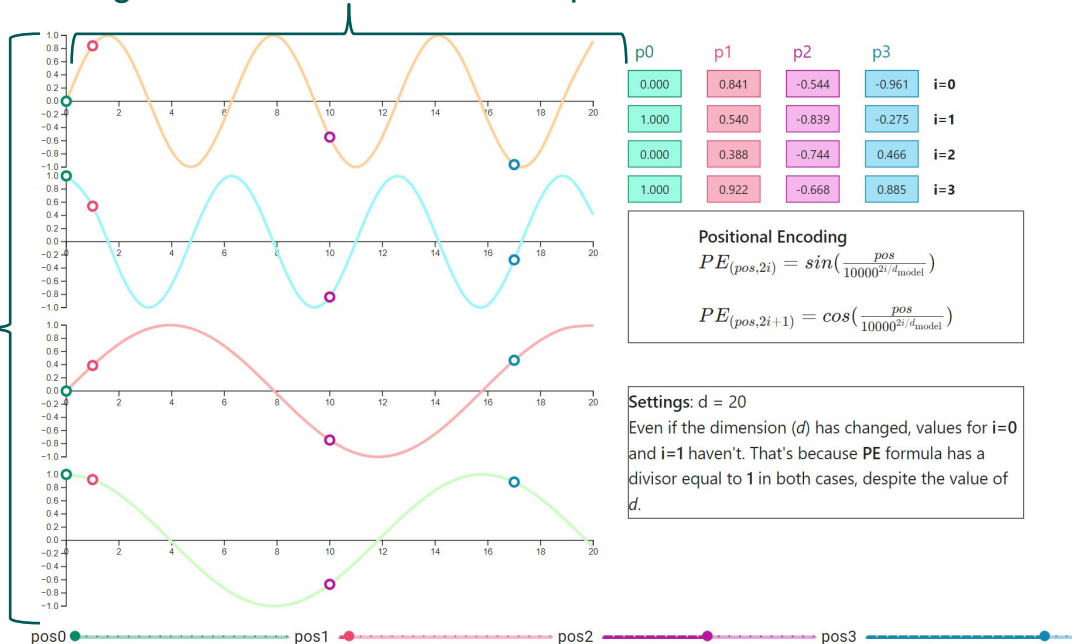
Sinusoid position embedding

$$p_{pos,2i} = \sin(pos/10000^{2i/d})$$

$$p_{pos,2i+1} = \cos(pos/10000^{2i/d})$$

Different wavelength at different embedding dimension to capture different relative positions

Adding different values based on positions



<https://erdem.pl/2021/05/understanding-positional-encoding-in-transformers>

Position Encoding: Absolute Position Information

Additive Position Encoding: add the positional information in the token embedding layer

$$\mathbf{x}' = \mathbf{x} + \mathbf{p}_{pos}$$

Position Embedding at position pos

Fully Learned Embeddings (e.g., in BERT)

$$\mathbf{p}_{pos} = \text{Embedding}(pos)$$

One embedding vector for each pos

Position Encoding: Absolute Position Information

Additive Position Encoding: add the positional information in the token embedding layer

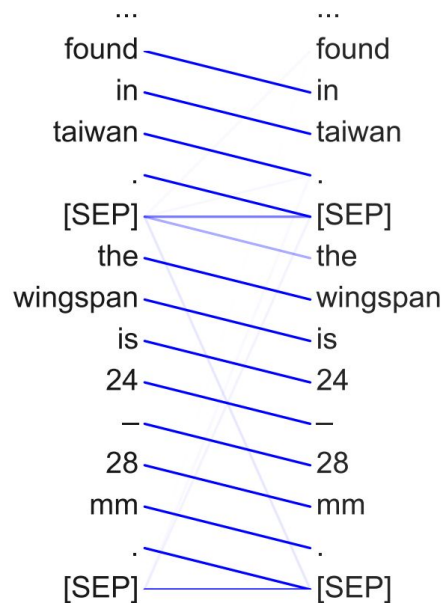
$$\mathbf{x}' = \mathbf{x} + \mathbf{p}_{pos}$$

Position Embedding at position pos

Fully Learned Embeddings (e.g., in BERT)

$$\mathbf{p}_{pos} = \text{Embedding}(pos)$$

One embedding vector for each pos



Learned some strong
position-based attention patterns [8]

Position Encoding: Relative Position Information

Language Prior: Only relative positions matters in language

I took my dog, Fido, to the park for his walk....

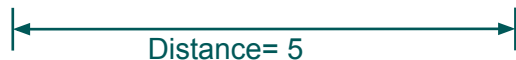


Blablabla...I took my dog, Fido, to the park for his walk....

Position Encoding: Relative Position Information

Language Prior: Only relative positions matters in language

I took my dog, Fido, to the park for his walk....



Blablabla...I took my dog, Fido, to the park for his walk....



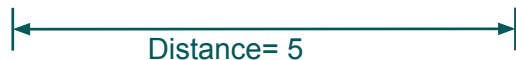
Encode relative position information in attention mechanism:

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$

Position Encoding: Relative Position Information

Language Prior: Only relative positions matters in language

I took my dog, Fido, to the park for his walk....



Blablabla...I took my dog, Fido, to the park for his walk....

Encode relative position information in attention mechanism:

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$

Using relative position embeddings [9]:

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}} + \underline{b_{i-j}}$$

Trainable Relative
Position Bias

Position Encoding: Relative Position Information

Language Prior: Only relative positions matters in language

I took my dog, Fido, to the park for his walk....



Blablabla...I took my dog, Fido, to the park for his walk....



Encode relative position information in attention mechanism:

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$

Using relative position embeddings [9]:

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}} + \underline{b_{i-j}}$$

Trainable Relative
Position Bias

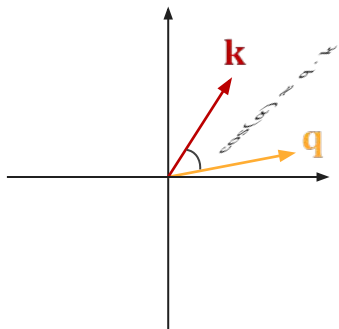
Model	Position Information	EN-DE BLEU	EN-FR BLEU
Transformer (base)	Absolute Position Representations	26.5	38.2
Transformer (base)	Relative Position Representations	26.8	38.7
Transformer (big)	Absolute Position Representations	27.9	41.2
Transformer (big)	Relative Position Representations	29.2	41.5

Performance of Relative Position Embedding on Machine Translation [10]

Position Encoding: Rotational Position Embedding

Geometry of dot product in attention mechanism

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$

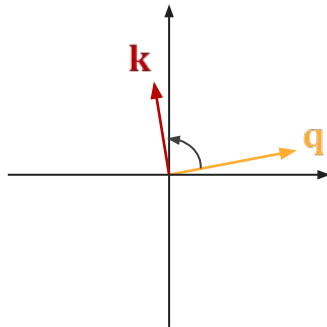
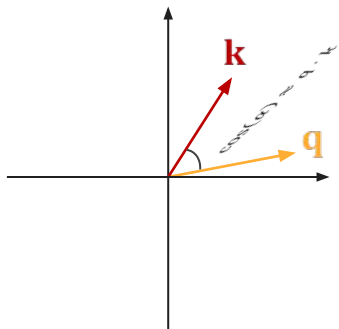


Attention score roughly as cosine of the vectors, assuming unit-lengths.

Position Encoding: Rotational Position Embedding

Geometry of dot product in attention mechanism

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$



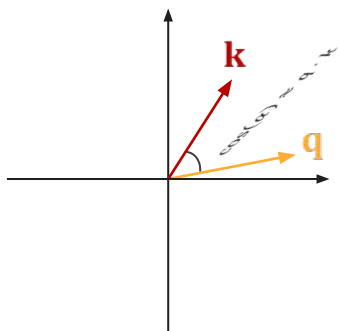
Attention score roughly as cosine of the vectors, assuming unit-lengths.

Lower attention importance if positions are far: rotating away

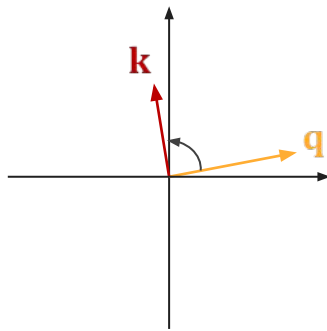
Position Encoding: Rotational Position Embedding

Geometry of dot product in attention mechanism

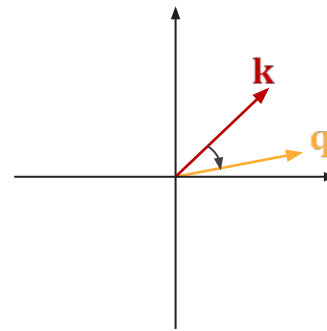
$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$



Attention score roughly as cosine of the vectors, assuming unit-lengths.



Lower attention importance if positions are far: rotating away

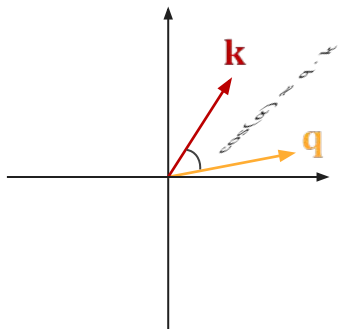


Higher attention importance if positions are close: rotating close

Position Encoding: Rotational Position Embedding

How to make the rotation only depend on relative positions?

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$

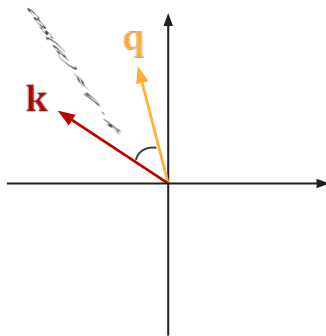
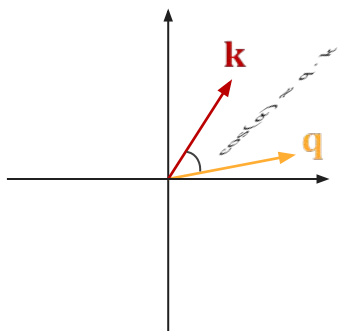


Attention score roughly as cosine of the vectors, assuming unit-lengths.

Position Encoding: Rotational Position Embedding

How to make the rotation only depend on relative positions?

$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$



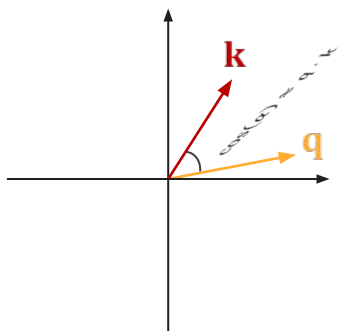
Attention score roughly as cosine of the vectors, assuming unit-lengths.

Rotating vectors together for the same disagrees based on positions changes

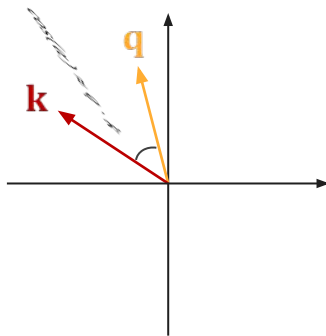
Position Encoding: Rotational Position Embedding

How to make the rotation only depend on relative positions?

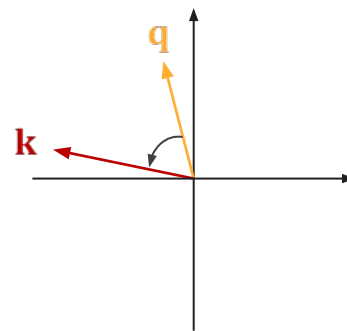
$$\text{Attention score}(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$



Attention score roughly as cosine of the vectors, assuming unit-lengths.



Rotating vectors together for the same disagrees based on positions changes

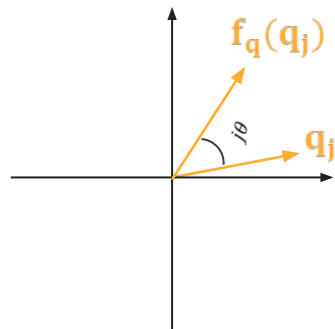


Position based prior holds the same at new absolute positions

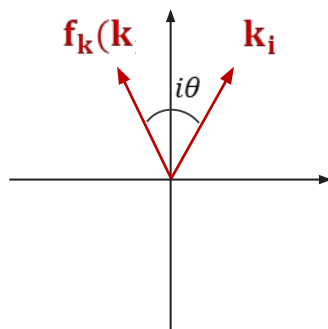
Position Encoding: Rotational Position Embedding

Incorporate the vector rotation in the attention mechanism (2d space) [11]:

$$f_q(q_j) = \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix} \begin{pmatrix} q_j^1 \\ q_j^2 \end{pmatrix} \quad f_k(k_i) = \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} \begin{pmatrix} k_i^1 \\ k_i^2 \end{pmatrix} \quad \theta_k = (1/10000)^{2(i-1)/d}$$



Rotate q by $j\theta$

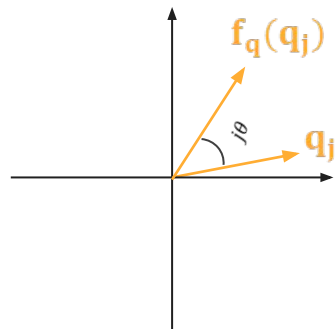


Rotate k by $i\theta$

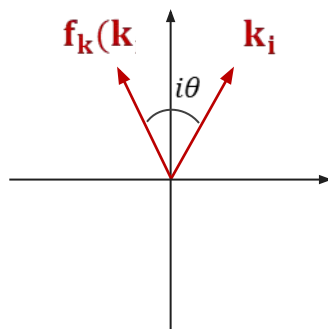
Position Encoding: Rotational Position Embedding

Incorporate the vector rotation in the attention mechanism (2d space) [11]:

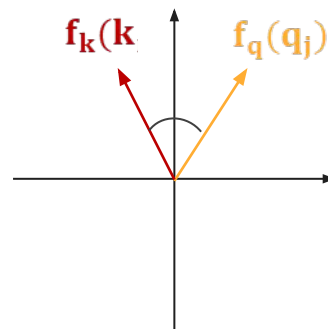
$$f_q(q_j) = \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix} \begin{pmatrix} q_j^1 \\ q_j^2 \end{pmatrix} \quad f_k(k_i) = \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} \begin{pmatrix} k_i^1 \\ k_i^2 \end{pmatrix} \quad \theta_k = (1/10000)^{2(i-1)/d}$$



Rotate q by $j\theta$



Rotate k by $i\theta$

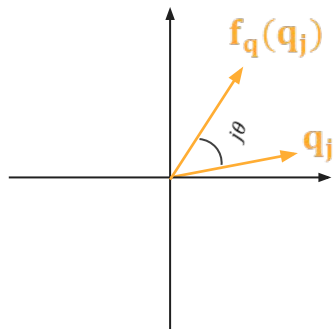


Attention only depends on $i - j$

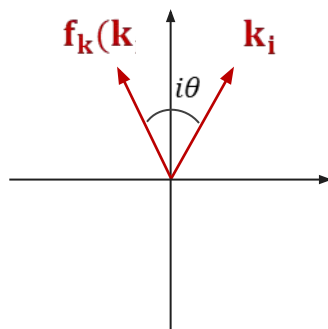
Position Encoding: Rotational Position Embedding

Incorporate the vector rotation in the attention mechanism (2d space) [11]:

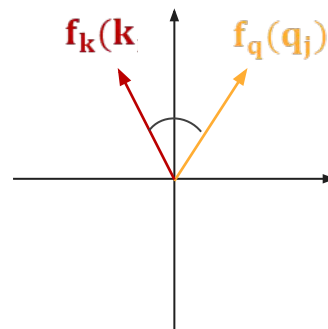
$$f_q(q_j) = \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix} \begin{pmatrix} q_j^1 \\ q_j^2 \end{pmatrix} \quad f_k(k_i) = \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} \begin{pmatrix} k_i^1 \\ k_i^2 \end{pmatrix} \quad \theta_k = (1/10000)^{2(i-1)/d}$$



Rotate q by $j\theta$



Rotate k by $i\theta$



Attention only depends on $i - j$

Attention score by the dot prod of rotated vectors:

$$\text{Attention score}(q_j, k_i) = f_q(q_j) \cdot f_k(k_i) = \begin{pmatrix} q_j^1 \\ q_j^2 \end{pmatrix}^T \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix}^T \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} \begin{pmatrix} k_i^1 \\ k_i^2 \end{pmatrix}$$

Position Encoding: Rotational Position Embedding

Full form in the high dimensional space [11]:

$$f_{q,k}(x_i) = \begin{pmatrix} \begin{pmatrix} \cos i\theta_1 & -\sin i\theta_1 \\ \sin i\theta_1 & \cos i\theta_1 \end{pmatrix} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \begin{pmatrix} \cos i\theta_{d/2} & -\sin i\theta_{d/2} \\ \sin i\theta_{d/2} & \cos i\theta_{d/2} \end{pmatrix} \end{pmatrix} \begin{pmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^{d/2} \\ x_i^{d/2} \end{pmatrix}$$

Partition dimensions into pairs and do the 2d rotation on each pair

With different wavelet lengths: $\theta_k = (1/10000)^{2(k-1)/d}$

Position Encoding: Rotational Position Embedding

Full form in the high dimensional space [11]:

$$f_{q,k}(x_i) = \begin{pmatrix} \begin{pmatrix} \cos i\theta_1 & -\sin i\theta_1 \\ \sin i\theta_1 & \cos i\theta_1 \end{pmatrix} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \begin{pmatrix} \cos i\theta_{d/2} & -\sin i\theta_{d/2} \\ \sin i\theta_{d/2} & \cos i\theta_{d/2} \end{pmatrix} \end{pmatrix} \begin{pmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^{d/2} \\ x_i^{d/2} \end{pmatrix}$$

Partition dimensions into pairs and do the 2d rotation on each pair

With different wavelet lengths: $\theta_k = (1/10000)^{2(k-1)/d}$

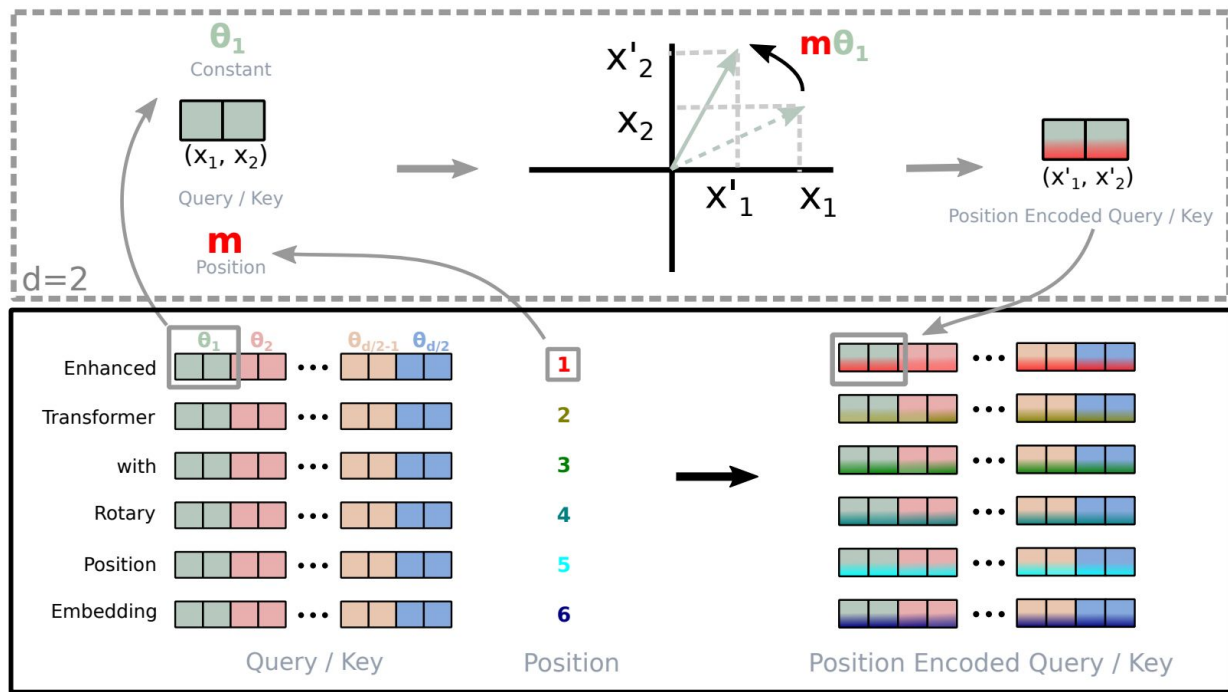
Rooted in the sinusoid absolute position embedding, but does multiplication (rotation):

$$\begin{aligned} \mathbf{x}' &= \mathbf{x} + \mathbf{p}_{pos} & p_{pos,2i} &= \sin(pos/10000^{2i/d}) \\ & & p_{pos,2i+1} &= \cos(pos/10000^{2i/d}) \end{aligned}$$

“We chose this function because we hypothesized it would allow the model to easily learn to attend by relative positions”---Transformer Paper

Position Encoding: Rotational Position Embedding

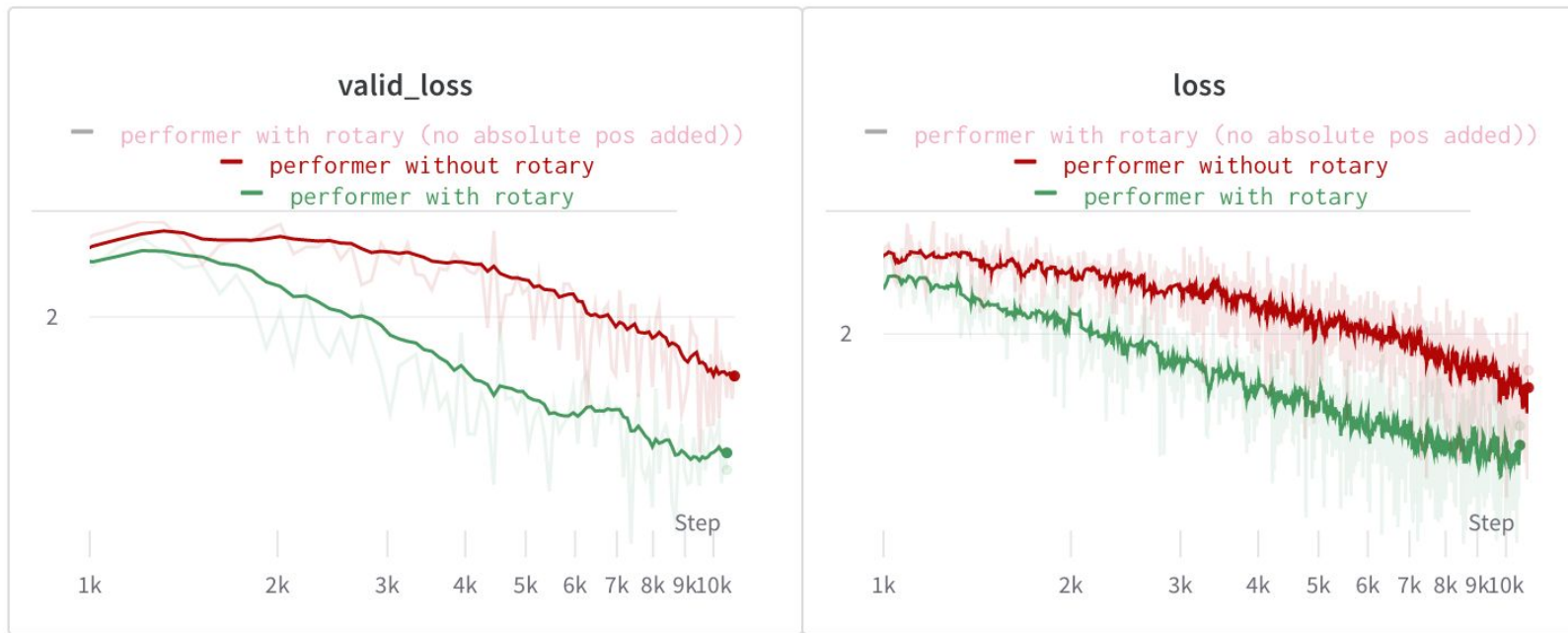
Putting it all together:



RoPE Embedding [11]

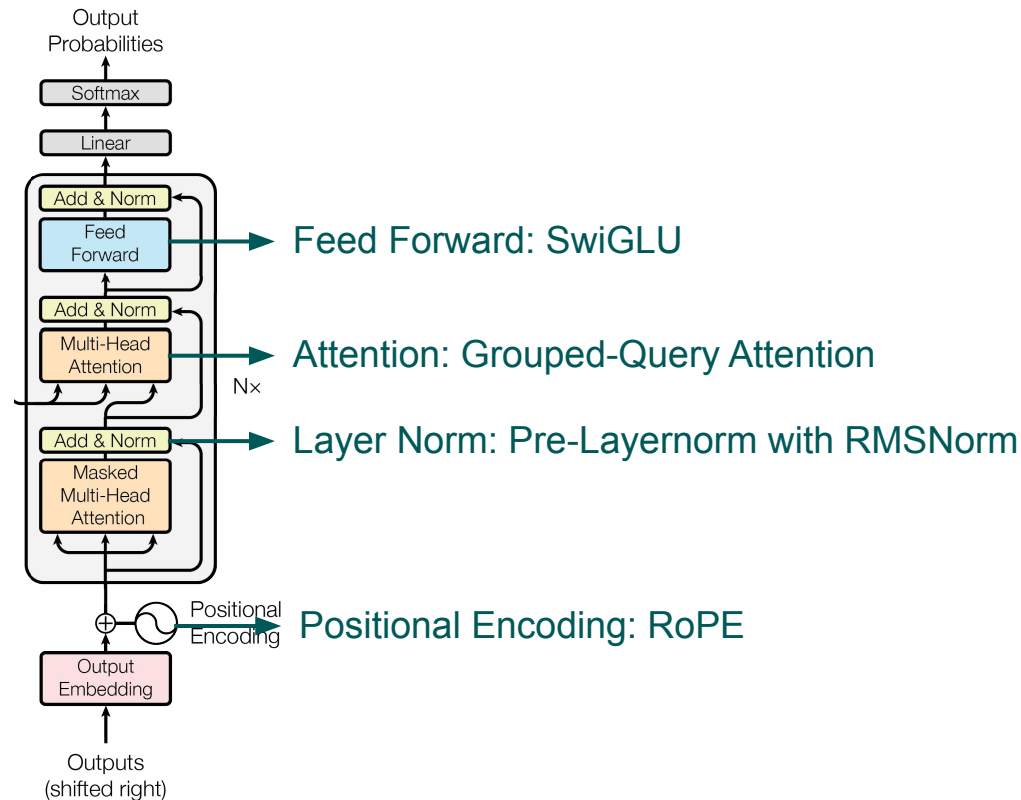
Position Encoding: Rotational Position Embedding

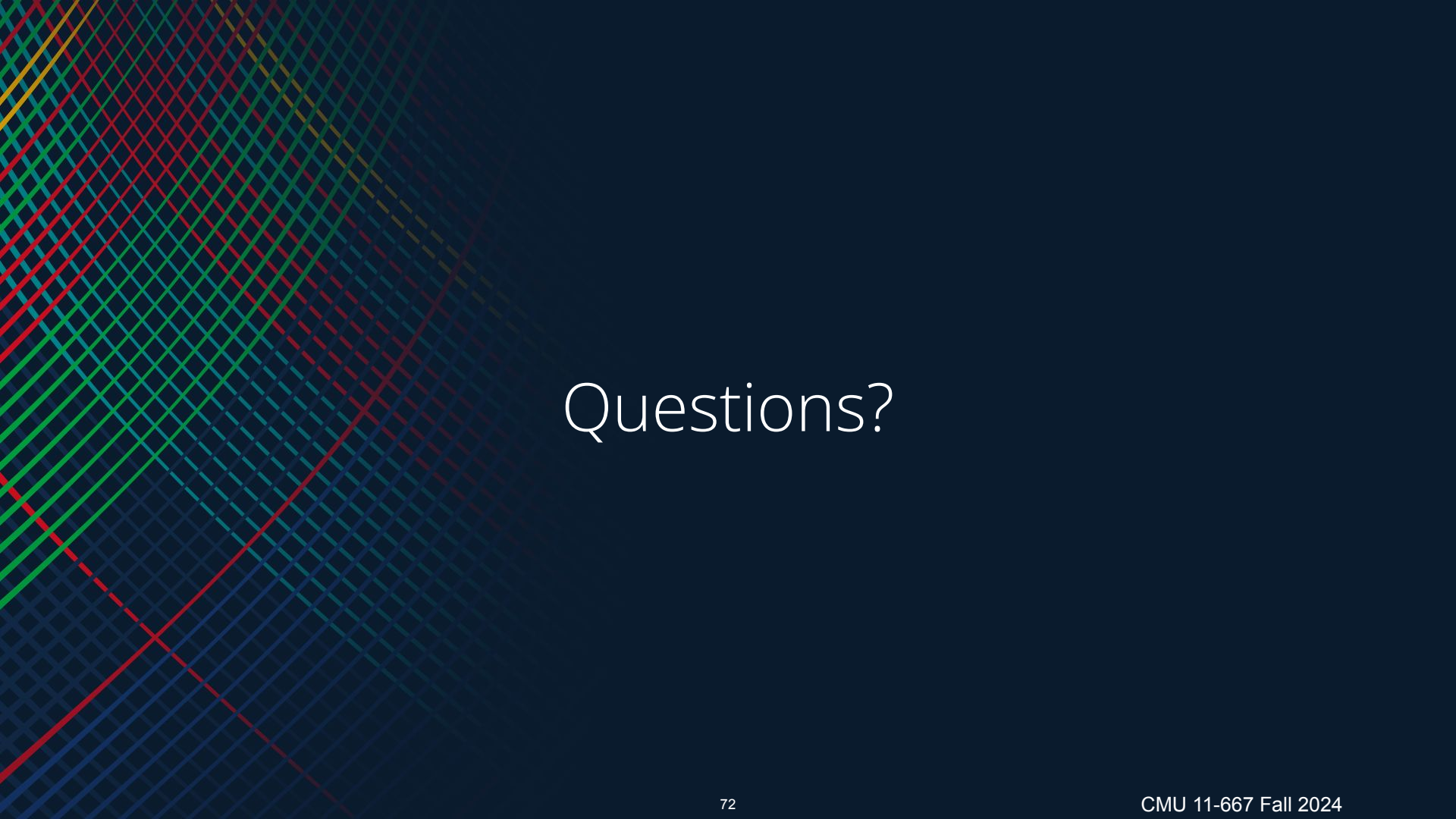
Performance with RoPE



<https://blog.eleuther.ai/rotary-embeddings/>

LLaMA3's Choice





Questions?